Lecture 2

Example 1.

Poker. classical probability.

$$\mathbb{P} = \frac{\text{number of certain outcomes}}{\text{total number of outcomes}}.$$

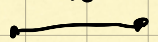
Example 2.

Buffon to compute π.

Geometric probability.

$$\mathbb{P} = \frac{\text{Area of a region}}{\text{Area of the total region}}.$$

l.



Geometric probability.

$$\frac{n}{N} \overset{?}{\approx} \frac{2l}{\pi \cdot a}$$

$$\pi \approx \frac{2lN}{a \cdot n}.$$

Example 3. flu test.

Bayes Theorem. $\Leftarrow$ $\begin{cases} \text{conditional probability.} \\ \text{Law of total probability.} \end{cases}$

Example 4. Random variable. $\begin{cases} \text{discrete r.v.} \\ \text{continuous r.v.} \end{cases}$

example 4.1. Toss a coin 3 times.

8 outcomes

Number of heads. is one example of random variable.

example. 4.2. Waiting a bus

A bus arrives at the bus station any time between 12:00 to 12:20.
bus waits for 5 mins.

A person arrives at the bus station any time between 12:00 to 12:20.

X = the number of minutes the person waits.

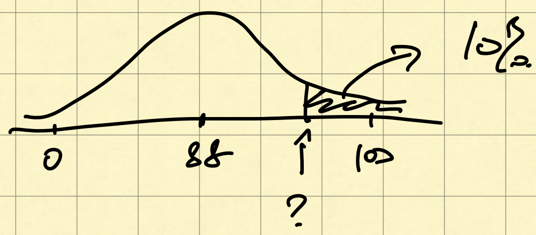
example 5. what score you need?

$\begin{cases} \underline{\text{normal distribution.}} \\ \text{Gamma distribution.} \\ \text{Beta distribution.} \end{cases}$

$\mu = 88$

$\sigma = \sqrt{15}$

$\underline{\mathcal{N}(\mu, \sigma^2)}$



Example 6. statistics

data $\quad X_1, X_2, \ldots, X_n$

Today: Set Theory.

__Def__ (set) A __set__ is a collection of distinct objects. The objects are its __elements__

example. $\{\text{"apple"}, \text{"orange"}, \text{"car"}\}$.

$\{0, 1, \pi, e, \frac{1}{2}\}$.

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

$[0, 1] = \{x \in \mathbb{R} \mid 0 \le x \le 1\}$.

empty set: a set with nothing in it, denoted by ϕ.

$\{\phi\}$ $\quad\quad \phi$

$\{1, \{1,2\}, \{1\}, \phi \}$.

Def (Subset). A is a subset of B, written $A \subset B$, if $\forall x \in A$, we have $x \in B$.

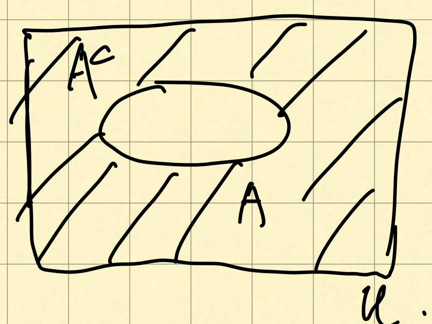
Def (Equality) We call $A = B$ if and only if $A \subset B$ and $B \subset A$.

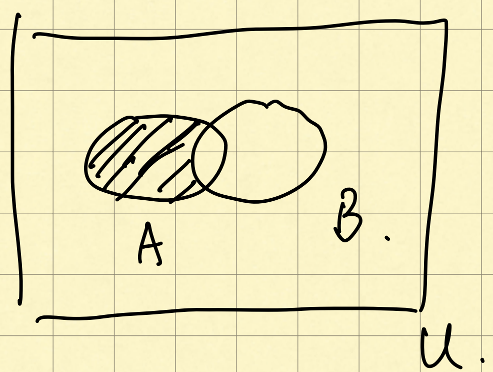
Def (complement, union, intersection, difference).

1. $A^c = \overline{A} := \{x \mid x \notin A\}$.
2. $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$.
3. $A \cap B := \{x \mid x \in A \text{ and } x \in B\}$.
4. $A \sim B := \{x \mid x \in A \text{ and } x \notin B\}$.

$$= \{x \mid \underline{x \in A} \text{ and } \underline{x \in B^c}\}.$$

$$= A \cap B^c$$





De Morgan's Rule.

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$$

$$(A \cup B \cap C)^c = A^c \cap B^c \cup C^c$$

Def (Cardinality) $|A|$ denotes the number of elements of A.

$|A| < \infty$ finite $\Leftrightarrow$ A contains finite number of elements

$|A| = \infty$ $\begin{cases} \text{countably infinite} & \mathbb{N},\ \mathbb{Z},\ \mathbb{Q}. \\ \text{uncountably infinite} & (0,1),\ \mathbb{R}. \ \text{unit square } \boxed{} \end{cases}$

Eg. $|\mathbb{Z}| = |\mathbb{Q}|$. $\quad \mathbb{Q} = \left\{ \frac{m}{n} \,\middle|\, m, n \in \mathbb{Z} \right\}$.

$$\begin{array}{cccccc}
\frac{0}{1}, & \frac{1}{1}, & \frac{2}{1}. & \cdots & & \\
\cancel{\frac{0}{2}}, & \frac{1}{2}, & \cancel{\frac{2}{2}}. & \frac{3}{2}. & \cdots & \\
\cancel{\frac{0}{3}}, & \frac{1}{3} & \frac{2}{3}, & \cancel{\frac{3}{3}}. & \frac{4}{3} & \cdots
\end{array}$$

$\mathbb{Z} \subset \mathbb{Q}$. $\Rightarrow$ $|\mathbb{Z}| \le |\mathbb{Q}|$.

$\mathbb{Z} \supset \mathbb{Q}$. X $|\mathbb{Z}| \ge |\mathbb{Q}|$? ✓

$$\boxed{\text{Claim : every rational number can be mapped into a unique integer number.}}$$

<u>Def</u> (Collection of sets).

$A \cup B$

$A \cap B$

- $A_0, A_1, A_2, A_3, \cdots$

$\{A_i\}_{i \in \mathbb{N}} = \{A_0, A_1, A_2, \cdots \}$.

- $A_a := \underbrace{(-a, a)}$.

$\{A_a\}_{a \in (0,1)}$.