Lecture 3.

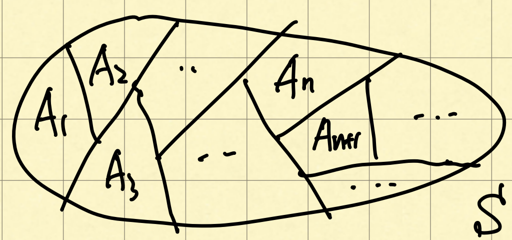
$\Gamma = \mathbb{N}$: $\{A_i\}_{i \in \Gamma} = \{A_1, A_2, \ldots, A_n, \ldots\}$

$\Gamma = (0, 1)$. $\{A_\alpha\}_{\alpha \in \Gamma}$ = an uncountable collection of open intervals.

Def (Pairwise disjoint). $\{A_i\}_{i \in \mathbb{N}}$ is pairwise disjoint if $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Def (Partition). $\{A_i\}_{i \in \mathbb{N}}$ is a partition of a set S if

- A_i are pairwise disjoint.
- $S = \bigcup_{i \in \mathbb{N}} A_i$.



Experiments. outcomes. events.

Def (Experiment). An experiment is a <u>repeatable</u> task with <u>well-defined</u> outcomes.

Def (Outcome)

outcomes are denoted $w_1, w_2, \ldots$

The set of all outcomes is called sample space S.

Def (Event). Any subset $E \subset S$ is an event.

We say E happens if the outcome of the experiment lies in E.

Example.

Experiment: Roll a six-sided die once.

outcomes: $1, 2, 3, 4, 5, 6$.

sample space: $S = \{1, 2, 3, 4, 5, 6\}$.

Events: "You got an even number".

$E \subset S$

$E = \{2, 4, 6\}$.

"You got number 10".

$E = \phi$.

"You got an integer number".

$E = S$.

Experiment = Roll a 6-sided die, then toss a coin.

$S = \{(1, H), (1, T), (2, H), (2, T) \ldots (6, H), (6, T)\}$.

$|S| = 12.$

Event = I got even number from the die.

$E = \{(2, H), (2,T), (4, H), (4,T), (6, H), (6, T)\}.$

$|S| < \infty.$

Example : Experiment =

Record the number of minutes till the first call arrives at the call center.

$S = \{0, 1, 2, 3, \ldots\}.$

Experiment =

Pick a point from the unit sphere.

$S = \{(x, y) \mid x^2 + y^2 \leq 1\}.$

E = point is within $\frac{1}{2}$ distance from the origin.

$E = \{(x,y) \mid \sqrt{x^2 + y^2} \leq \frac{1}{2}\}.$

E^c

Example. Experiment : you order fries and a pizza from Ubereats. Let x be the delivery time of fries, and y be the delivery time of the pizza.

$$S = \{(x, y) \mid x \geq 0,\ y \geq 0\}.$$

$$A = \{(x, y) \in S \mid x \leq 10\}.$$

$$B = \{(x, y) \in S \mid y \leq 15\}.$$

$A \cup B$ = at least one item arrive within 15 mins
= I will have something to eat in 15 mins.
= ...

$(A \cap B)^c$, $A \setminus B$.

What is probability?

Classical probability.

Roll a fair 6-sided die once.

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Assign each outcome $w_i \in S$ a number.

$$\mathbb{P}(1) = \mathbb{P}(2) \cdots = \mathbb{P}(6) = \frac{1}{6}.$$

- $|S| < \infty$.

- $\forall w \in S$, $\mathbb{P}(w) = \dfrac{1}{|S|}$.

we defined it.

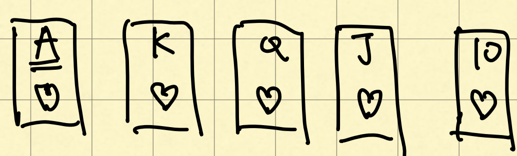
Def (Classical probability). Let $S = \{w_1, \dots, w_n\}$ be a finite sample space. All outcomes have the same probability defined as $\mathbb{P}(w_i) = \frac{1}{n}$.
For an even $A \subset S$ containing m outcomes,

$$\mathbb{P}(A) := \frac{m}{n} = \frac{|A|}{|S|}.$$

definition is trivial, but computing can be tricky.

① $|S| = ?$
② $|A| = ?$ $\Rightarrow$ $\mathbb{P}(A) = \dfrac{|A|}{|S|}$ ✓.

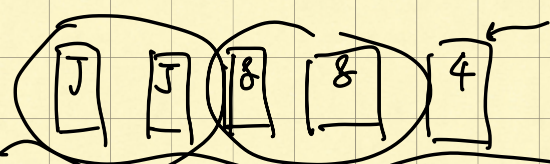
But ② can be tricky.



2, 10, J Q. K. A.

♢ ♡ ♧ ♤

$\mathbb{P}$(Royal ~~straight~~ flush) $= \dfrac{4}{|S|}$.



$\mathbb{P}$(Two pairs) $= \dfrac{?}{|S|}$

Counting.

Combinatorics.

How to count ?

- (Principle of Addition) If a task can be done in m way or in n ways. then the total ways is $m + n$.

- (principle of multiplication) If a task can be done is Two steps: the first step m way. the second step. n ways. then the total way is $m * n$.

Question:

You have n distinct objects. select k from them.

How many ways of selection ?

- Does the order matter ?
- Is the selection with or without replacement ?

{apple, orange, banana}. Select 2 from them.

order matters: apple, orange. $\neq$ orange, apple.

order not matters: ... = ...

with replacement: apple, apple

without replacement :

Case I: order matters without replacement.

selection can be built in k steps:

step 1: select 1st object : n ways.

step 2 : 2nd : $n-1$ ways.

$\vdots$ $\vdots$

step k : k-th : $n-k+1$ ways.

$n \cdot (n-1) \cdots (n-k+1)$ ways.