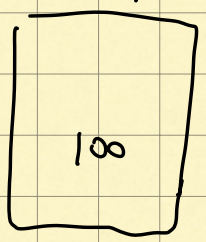
120 pt.

Minimum	—
Median	85.5
Maximum	119.5
Average	81

100 pt.



Average < 81.

$aX + b$

Average $\to 83$

(Binomial Distribution.)

X = number of S among n trials.

$$\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

(Hypergeometric Distribution)

N objects. M good $N-M$ bad.

select n objects all at once. $\xi :=$ the number of good objects selected.

$$\mathbb{P}\{\xi = k\} = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}.$$

$$p := \frac{M}{N}.$$

$$E(\xi) = n \cdot p \qquad Var(\xi) = \left(\frac{N-n}{N-1}\right) np(1-p).$$

$\left(\frac{N-n}{N-1}\right) < 1$

$$\mathbb{P}\{\xi_{hyp} = k\} \overset{n << N}{\approx} \mathbb{P}\{\xi_{bin} = k\}.$$

(Negative Binomial Distribution).

A sequence of independent Bernoulli trial.

$SFFS\cdots$

Repeat the trial until total number of (r) success.

$\xi :=$ the number of F until r S's.

$\xi \sim \text{NegBinomial}(r, p)$.

$$\mathbb{P}\{\xi = k\} = \binom{r-1+k}{k}(1-p)^k p^r$$

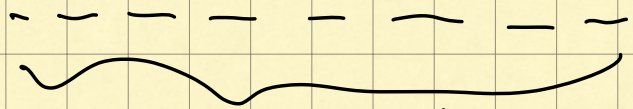
$r = 1$.

$\mathbb{P}\{\xi = k\} = (1-p)^k p$.



r-th success.

(r-1) S.
k F.



r-1 + k k F
r-1 S

$$E(\xi) = r\left(\frac{1}{p} - 1\right).$$

$Var(X_1 + \cdots + X_n)$

$$\mathrm{Var}(\xi) = r\,\frac{\left(\frac{1}{p}-1\right)}{p}. \qquad = \mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_n)$$

Poisson distribution.

Def. (Poisson distribution).

ξ . $\mathbb{P}\{\xi = k\} = \frac{\mu^k}{k!} e^{-\mu}$, $k = 0, 1, 2, \ldots$

Lemma. (Poisson Limit). If $n \to \infty$, $p \to 0$, $np = \mu$, then for all $k = 0, 1, 2, \ldots$

$$\binom{n}{k} p^k (1-p)^{n-k} \longrightarrow \frac{\mu^x}{x!} e^{-\mu}.$$

Def (Poisson distribution). is defined as the limit of Binomial Distribution.

The Poisson Process.

Think of phone calls arriving at call center.

- In a short time interval Δt, the probability of exactly one call arrives is

$$\alpha \Delta t + o(\Delta t). \qquad \frac{o(\Delta t)}{\Delta t} \approx 0.$$

- the probability that two or more calls arrive in Δt is $o(\Delta t)$.

- independence.

Let $P_k(t)$ denote the probability that exactly k calls arrive in time interval t.

Then $$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}.$$

which is Poisson (μ), $\mu = \lambda t$.

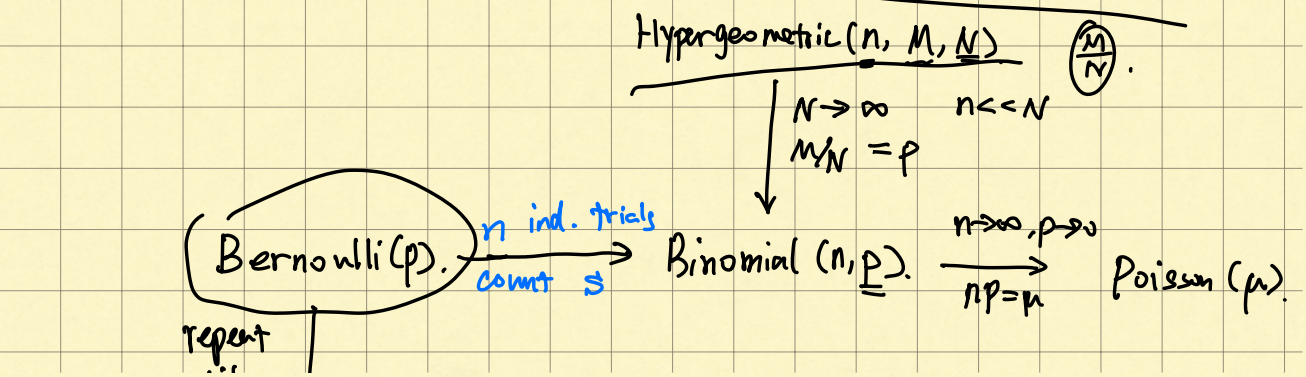
(Thinning of Poisson process).

$N \sim$ Poisson (10). be the number of arrivals in the hour.

$N = n$, the number of purchases $\sim$ Bin$(n, 0.3)$.

The number of purchases made in the hour?

poisson (10×0.3).



until first S

Geo(p). —repeat until r S→ Neg Binomial (r, p)

	Support k	pmf $\mathbb{P}\{\xi = k\}$	$E[\xi]$	$Var(\xi)$
Bernoulli(p)	$\{0, 1\}$	$p^k(1-p)^{1-k}$	p	$p(1-p)$.
Binomial(p)	$\{0, \dots, n\}$	$\binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$.
Hypergeo(n, M, N)	k feasible	$\frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$	np	$\frac{N-n}{N-1} np(1-p)$.
Geo(p)	$\{1, 2, \dots\}$	$(1-p)^{k-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
NegBin(r, p)	$\{0, 1, \dots\}$	$\binom{k+r-1}{k} p^r (1-p)^k$	$r(\frac{1}{p}-1)$	$\frac{r(1-p)}{p^2}$.
Poisson(μ)	$\{0, 1, \dots\}$	$\frac{\mu^k}{k!} e^{-\mu}$	μ	μ.