Chapter 11

Review for Midterm 1

Lecture 11 | Devore 9e: Ch. 2–3.5

Midterm 1 covers Lectures 1–10: probability spaces, counting, conditional probability, independence, Bayes, and discrete random variables through variance.

11.1 What you must be able to do

1. **Write down a sample space** and express a verbal event as a subset (Lecture 2). Translate "at least one", "none", "exactly two" into unions, complements, intersections.
2. **Count** (Lecture 3): decide *order?* and *replacement?*, then use the right entry of the table.
3. **Compute geometric probabilities** as ratios of length/area (Lecture 4).
4. **Use the probability axioms** (Lecture 4) to prove small identities: $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$, $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$, inclusion–exclusion.
5. **Condition** (Lectures 5–7): multiplication rule for sequential experiments, law of total probability when there is a hidden case split, Bayes when the question reverses the given.
6. **Check independence** from the definition $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$ — and know that pairwise does not imply mutual (Lectures 5–6).
7. **Work with a discrete random variable** (Lectures 8–10): find the pmf, build the cdf, compute $\mathbb{E}[X]$, $\mathbb{E}[h(X)]$, and $\mathrm{Var}(X)$.
8. **Recognize the named families** so far: Bernoulli (the indicator variable of Lecture 8), binomial, geometric, hypergeometric.

11.2 Formula sheet

Counting

	without replacement	with replacement
order matters	$A_n^k = \frac{n!}{(n-k)!}$	n^k
order doesn't matter	$\binom{n}{k}$	$\binom{n+k-1}{n-1}$

Probability

$$\mathbb{P}(A^c) = 1 - \mathbb{P}(A) \qquad \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \qquad \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B \mid A)$$

$$\mathbb{P}(B) = \sum_i \mathbb{P}(A_i)\mathbb{P}(B \mid A_i) \quad \mathbb{P}(A_i \mid B) = \frac{\mathbb{P}(A_i)\mathbb{P}(B \mid A_i)}{\sum_j \mathbb{P}(A_j)\mathbb{P}(B \mid A_j)}$$

$A \perp\!\!\!\perp B \iff \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$.

Discrete random variables

$$F_X(x) = \mathbb{P}\{X \le x\}, \qquad \mathbb{E}[X] = \sum_i x_i p_i, \qquad \mathbb{E}[h(X)] = \sum_i h(x_i)p_i,$$

$$\operatorname{Var}(X) = \mathbb{E}\big[(X-\mu)^2\big] = \mathbb{E}[X^2] - \mathbb{E}[X]^2, \qquad \sigma_X = \sqrt{\operatorname{Var}(X)},$$

$$\mathbb{E}[aX+b] = a\mathbb{E}[X] + b, \qquad \operatorname{Var}(aX+b) = a^2 \operatorname{Var}(X), \qquad \sigma_{aX+b} = |a|\,\sigma_X.$$

Families so far

Name	pmf	support	$\mathbb{E}$	Var
Bernoulli(p)	$p^k(1-p)^{1-k}$	$k \in \{0,1\}$	p	$p(1-p)$
Binomial(n,p)	$\binom{n}{k}p^k(1-p)^{n-k}$	$k = 0,\ldots,n$	np	$np(1-p)$
Geometric(p)	$(1-p)^{k-1}p$	$k = 1,2,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric(n,M,N)	$\frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$	k feasible	$n\frac{M}{N}$	$\frac{N-n}{N-1}n\frac{M}{N}\left(1-\frac{M}{N}\right)$

11.3 The three mistakes that cost the most points

1. **Confusing $\mathbb{P}(A \mid B)$ with $\mathbb{P}(B \mid A)$.** Read the sentence again: what is *given*, and what is being *asked*? The disease example (Lecture 7) exists entirely to make this vivid.
2. **Treating disjoint as independent.** Disjoint events with positive probability are maximally *dependent*. "Mutually exclusive" is a statement about $A \cap B$; "independent" is a statement about $\mathbb{P}(A \cap B)$.
3. **Forgetting that $\mathbb{E}[h(X)] \neq h(\mathbb{E}[X])$.** In particular $\mathbb{E}[X^2] \neq \mathbb{E}[X]^2$; their difference is the variance.

Remark 11.1. Practice problems for this midterm are in `exams/2026-spring/tex/` — see `practice1.tex` and `make-up-exam.tex`, both with the same topic coverage. (`mid2.tex` and `mid2-makeup.tex` are Midterm 2 material.)