Chapter 17

Review for Midterm 2

Lecture 17 | Devore 9e: Ch. 3.5–4.5

Midterm 2 covers Lectures 12–16 — the remaining discrete families, continuous random variables, and the named continuous distributions — together with the hypergeometric distribution from Lecture 10, since Devore 3.5 treats it alongside the negative binomial. Note that joint distributions are *not* on this exam — they start next lecture.

17.1 What you must be able to do

1. **Change of variable.** Given X with known cdf/pdf and $Y = g(X)$, find the cdf and then the pdf of Y. Always go through the cdf; always state the new range.

$$F_Y(y) = \mathbb{P}\{Y \le y\} = \mathbb{P}\{g(X) \le y\} \longrightarrow f_Y(y) = F_Y'(y).$$

2. **Use a pdf.** Compute $\mathbb{P}\{a \le X \le b\} = \int_a^b f$, normalize an unknown constant via $\int f = 1$, and find $\mathbb{E}[X]$, $\mathbb{E}[h(X)]$, $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.
3. **Normal distribution.** Standardize, use the z-table for both probabilities and percentiles, exploit symmetry ($\Phi(-z) = 1 - \Phi(z)$).
4. **Derive** the negative binomial (Lecture 12) and hypergeometric (Lecture 10) pmfs — not just quote them. Both are counting arguments.
5. **Poisson process.** Know which of Poisson / exponential / gamma answers which question:

how many events in time t?	$\mathrm{Poisson}(\lambda t)$
how long until the 1st event?	$\mathrm{Exponential}(\lambda) = \mathrm{Gamma}(1, 1/\lambda)$
how long until the α-th event?	$\mathrm{Gamma}(\alpha, 1/\lambda)$

17.2 Formula sheet

Continuous random variables

$$F(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t, \qquad f(x) = F'(x), \qquad \mathbb{E}[h(X)] = \int_{-\infty}^{\infty} h(x)f(x)\,\mathrm{d}x,$$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mu^2, \qquad f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\tfrac{d}{dy}g^{-1}(y)\right|.$$

Named continuous families

	pdf	$\mathbb{E}$, Var
Uniform$[A, B]$	$\frac{1}{B-A}$ on $[A, B]$	$\frac{A+B}{2}$, $\frac{(B-A)^2}{12}$
Normal(μ, σ^2)	$\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$	μ, σ^2
Exponential(λ)	$\lambda e^{-\lambda x}$, $x \geq 0$	$\frac{1}{\lambda}$, $\frac{1}{\lambda^2}$
Gamma(α, β)	$\frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}$, $x \geq 0$	$\alpha\beta$, $\alpha\beta^2$
$\chi^2_k = \text{Gamma}(\frac{k}{2}, 2)$	—	k, $2k$
Weibull(α, β)	$\frac{\alpha}{\beta}(\frac{x}{\beta})^{\alpha-1}e^{-(x/\beta)^\alpha}$	$\beta\Gamma(1+\frac{1}{\alpha})$, $\beta^2[\Gamma(1+\frac{2}{\alpha}) - \Gamma(1+\frac{1}{\alpha})^2]$
Lognormal(μ, σ^2)	$\frac{1}{\sqrt{2\pi}\sigma x}e^{-(\ln x-\mu)^2/2\sigma^2}$	$e^{\mu+\sigma^2/2}$, $(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Beta(α, β)	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$, $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Facts worth quoting

$$F_{\text{Exponential}}(x) = 1 - e^{-\lambda x}, \qquad F_{\text{Weibull}}(x) = 1 - e^{-(x/\beta)^\alpha},$$

memorylessness: $\mathbb{P}\{X > s + t \mid X > s\} = \mathbb{P}\{X > t\}$ for the exponential only.

17.3 The three mistakes that cost the most points

1. **Treating $f(x)$ as a probability.** It is a density; it may exceed 1, and $\mathbb{P}\{X = x\} = 0$ always.
2. **Losing the range in a change of variable.** A correct algebraic formula on the wrong interval is worth very little. Track where g maps the support of X.
3. **Mixing up λ and β.** The exponential is often written with rate λ; the gamma with scale $\beta = 1/\lambda$. Write down which convention the problem uses before computing anything.

Remark 17.1. Practice problems: `exams/2026-spring/practice2.tex`, and the Fall and Spring Midterm 2 papers in `exams/`.